Solución

① El problema indica que es gamma, así que procedemos a buscar los parametros:

$E[X] = 10$ $\qquad E[x] = \frac{\alpha}{\lambda} = 10 \qquad 10\lambda = \alpha \quad \alpha = \frac{10}{5} = 2$

$VAR = 50$

$VAR(x) = \frac{\alpha}{\lambda^2} = 50 \qquad 50 = \frac{10\cancel{\lambda}}{\lambda^{\cancel{2}}} = \lambda = \frac{10}{50}$

a) $\mathbb{P}[X \leq 50] = \int_0^{50} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx \qquad \lambda = \frac{1}{5}$

A

X se definio como la prob. de la duración funcional

$$\mathbb{P}[x \leq 50] = \int_0^{50} \left(\frac{1}{5}\right)^2 \frac{x^{2-1} e^{-x/5}}{\Gamma(2)} dx = \frac{-xe^{-x/5}}{5} - e^{-x/5} \Big|_0^{50}$$

$$= -\frac{50e^{-50/5}}{5} - e^{-50/5} + \frac{\cancel{0}e^{-0/5}}{5} + e^{-0/5}$$

$$= 1 - 11e^{-10} \approx \underline{0.99950}$$

b) $\mathbb{P}[X > 10] = 1 - \int_0^{10} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx$

A

X = Funcione

La reducción me pide la prob de que no funcione las primeras 10 semanas, eso debe ser 1 − P[que funcione 10 o mas semanas]

$$\mathbb{P}[x > 10] = 1 - \int_0^{10} \left(\frac{1}{5}\right)^2 \frac{x^{2-1} e^{-x/5}}{\Gamma(2)} dx = 1 - \left(-\frac{xe^{-x/5}}{5} - e^{-x/5} \Big|_0^{10}\right)$$

$$= 1 - \left(-\frac{10e^{-10/5}}{5} - e^{-10/5} + \frac{\cancel{0}e^{-0/5}}{5} + e^{-0/5}\right)$$

$$= 1 - (1 - 3e^{-2}) = 1 - 0.5939 = \underline{0.4061}$$

② El problema ya indica que es dist. gamma con parametros

$\lambda = 2$
$\alpha = 3$

Nos pide la prob. de que la planta no sea suficiente en un dia, eso significa que la ciudad requiera más de 12 millones k/h.

$$P[x > 12] = 1 - \int_0^{12} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx$$

$$= 1 - \int_0^{12} \frac{(2)^3 x^{3-1} e^{-2x}}{\Gamma(3)} dx =$$

$$= 1 - \left(-2x^2 e^{-2x} - 2x e^{-2x} - e^{-2x} \Big|_0^{12}\right)$$

$$= 1 - \left(1 - 313 e^{-24}\right) \approx 1.1816 \times 10^{-8} \approx \emptyset$$

③ R1: Falta el valor esperado lo sacamos con M_1 y $f_1(t)$

$$M_1 = \int_0^\infty t \cdot \lambda e^{-\lambda t} dt = -te^{-\lambda t} - \frac{e^{-\lambda t}}{\lambda} \Big|_0^\infty =$$

dt	$\int dv$
t +	$\lambda e^{-\lambda t}$
1 −	$-e^{-\lambda t}$
$\emptyset$	$e^{-\lambda t}/\lambda$

$$= -\infty \cancel{e^{-\lambda\infty}}^{0} - \cancel{\frac{e^{-\lambda\infty}}{\lambda}}^{\emptyset} + \cancel{\emptyset e^{-\lambda\emptyset}}^{0} + \cancel{\frac{e^{-\emptyset\lambda}}{\lambda}}^{1}$$

$$= \frac{1}{\lambda}$$

R2: Falta la var; $M_2 - M_1^2$ con $f_2(t)$ =

$$M_1 = \int_0^\infty t \cdot \lambda^2 t e^{-\lambda t} dt = -t^2 \lambda e^{-\lambda t} - 2te^{-\lambda t} - \frac{2e^{-\lambda t}}{\lambda} \Big|_0^\infty$$

dt	$\int dv$
t^2 +	$\lambda^2 e^{-\lambda t}$
$2t$ −	$-\lambda e^{-\lambda t}$
2 +	$e^{-\lambda t}$
$\emptyset$	$-e^{-\lambda t}/\lambda$

$$= -\infty^2 \cancel{\lambda e^{-\lambda\infty}}^{\emptyset} - \cancel{2\infty e^{-\lambda\infty}}^{\emptyset} - \cancel{\frac{2e^{-\lambda\infty}}{\lambda}}^{\emptyset} + \emptyset + \emptyset + \cancel{\frac{2e^{-\lambda\emptyset}}{\lambda}}$$

$$= \frac{2}{\lambda}$$

$$M_2 = \int_0^\infty t^2 \lambda^2 t e^{-\lambda t} dt =$$

dt	$\int dv$
t^3 +	$\lambda^2 e^{-\lambda t}$
$3t^2$ −	$-\lambda e^{-\lambda t}$
$6t$ +	$e^{-\lambda t}$
6 −	$e^{-\lambda t}/\lambda$
$\emptyset$	$e^{-\lambda t}/\lambda^2$

$$M_2 = -t^3 \lambda e^{-\lambda t} - 3t^2 e^{-\lambda t} - \frac{6te^{-\lambda t}}{\lambda} - \frac{6e^{-\lambda t}}{\lambda^2} \Bigg|_0^\infty$$

$$M_2 = -\emptyset - \emptyset - \emptyset - \emptyset + \emptyset + \emptyset + \emptyset + \frac{6}{\lambda^2}$$

$$M_2 = \frac{6}{\lambda^2}$$

$$VAR = \frac{6}{\lambda^2} - \left(\frac{2^2}{\lambda^2}\right) = \frac{6-4}{\lambda^2} = \underline{\underline{\frac{2}{\lambda^2}}}$$

R3: Falta el Pdf de 3 vars. Hay que hacer una convolución.

$$f_3(t) = \int_0^t f_2(t_1) \cdot f_1(t_2)\, dt_1$$

sabemos que $t = t_1 + t_2$

despejamos para t_2 y sustituimos

$$f_3(t) = \int_0^t f_2(t_1) \cdot f_1(t - t_1)\, dt_1$$

$$= \int_0^t \lambda^2 t_1 e^{-\lambda t_1} \lambda e^{-\lambda(t-t_1)}\, dt_1$$

$$= \lambda^3 e^{-\lambda t} \int_0^t t_1 \cancel{e^{-\lambda t_1}} \cancel{e^{+\lambda t_1}}\, dt_1 = \lambda^3 e^{-\lambda t} \int_0^t t_1\, dt_1$$

$$= \lambda^3 e^{-\lambda t} \frac{t_1^2}{2}\Bigg|_0^t = \frac{\lambda^3 t^2 e^{-\lambda t}}{2} - \cancel{\frac{\lambda^3 \emptyset^2 e^{-\lambda \emptyset}}{2}}$$

$$f_3(t) = \frac{\lambda^3 t^2 e^{-\lambda t}}{2}$$

R4: Falta el acumulado de 4, solo integramos PDF_4 de 0 a t.

$$F_4(t) = \int_0^t \frac{\lambda^4 t^3 e^{-\lambda t}}{6}\, dt =$$

dt	$\int dv$
t^3 +	$\lambda^4 e^{-\lambda t}$
$3t^2$ -	$-\lambda^3 e^{-\lambda t}$
$6t$ +	$\lambda^2 e^{-\lambda t}$
6 -	$-\lambda e^{-\lambda t}$
$\emptyset$	$e^{-\lambda t}$

$$F(t) = \frac{1}{6}\left(-t^3\lambda^3 e^{-\lambda t} - 3t^2\lambda^2 e^{-\lambda t} - 6t\lambda e^{-\lambda t} - 6e^{-\lambda t}\Big|_0^t\right)$$

$$F(t) = -\frac{t^3\lambda^3 e^{-\lambda t}}{6} - \frac{t^2\lambda^2 e^{-\lambda t}}{2} - t\lambda e^{-\lambda t} - e^{-\lambda t}$$

$$+ \emptyset + \emptyset + \emptyset + 1$$

$$F(t) = 1 - \frac{(t\lambda)^3 e^{-\lambda t}}{6} - \frac{(t\lambda)^2 e^{-\lambda t}}{2} - t\lambda e^{-\lambda t} - e^{-\lambda t} \quad //$$

(4) Para demostrar σ, hay que demostrar la VAR. Entonces

$$M_1 = \int_0^\infty x \cdot f(x)\, dx = \int_0^\infty x \cdot \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}\, dx$$

$$= \int_0^\infty \frac{(x\lambda)^\alpha e^{-\lambda x}}{\Gamma(\alpha)}\, dx = \frac{1}{\Gamma(\alpha)} \int_0^\infty (x\lambda)^\alpha e^{-\lambda x}\, dx$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty y^\alpha e^{-y} \frac{dy}{\lambda} = \frac{1}{\lambda\Gamma(\alpha)} \cdot \Gamma(\alpha+1) = \frac{\alpha\cancel{\Gamma(\alpha)}}{\lambda\cancel{\Gamma(\alpha)}} = \frac{\alpha}{\lambda}$$

$$M_2 = \int_0^\infty x^2 \cdot f(x)\, dx = \int_0^\infty \frac{\lambda^\alpha x^{\alpha+1} e^{-\lambda x}}{\Gamma(\alpha)}\, dx = \frac{1}{\Gamma(\alpha)} \int_0^\infty \frac{\lambda}{\lambda} \lambda^\alpha x^{\alpha+1} e^{-\lambda x} dx$$

$$= \frac{1}{\Gamma(\alpha)\lambda} \int_0^\infty (\lambda x)^{\alpha+1} e^{-\lambda x}\, dx = \frac{1}{\Gamma(\alpha)\lambda} \int_0^\infty y^{\alpha+1} e^{-y} \frac{dy}{\lambda} = \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)\lambda^2}$$

$$= \frac{(\alpha+1)\Gamma(\alpha+1)}{\lambda^2\Gamma(\alpha)} = \frac{(\alpha+1)\alpha\cancel{\Gamma(\alpha)}}{\lambda^2\cancel{\Gamma(\alpha)}} = \frac{\alpha^2+\alpha}{\lambda^2}$$

$$VAR = \frac{\alpha^2+\alpha}{\lambda^2} - \frac{\alpha^2}{\lambda^2} = \frac{\alpha}{\lambda^2} \qquad \sigma = \sqrt{VAR}$$

$$\sigma = \sqrt{\alpha} / \lambda \quad // \text{ QED.}$$